The One-World Interpretation of Quantum Mechanics

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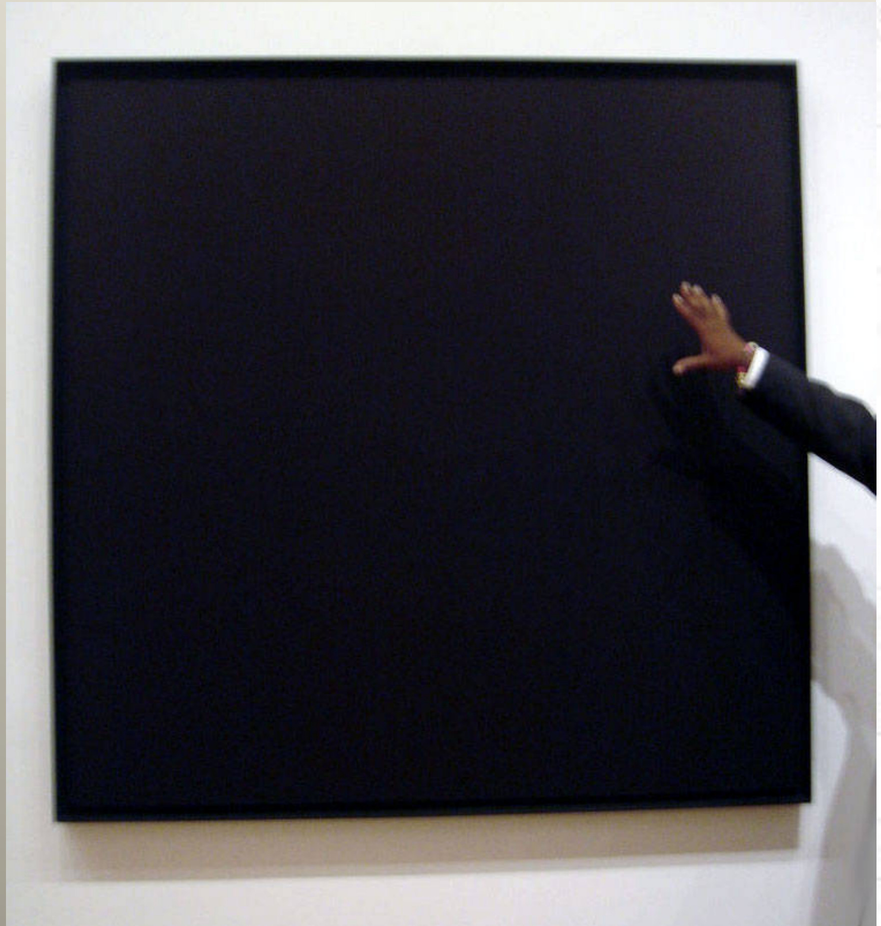
Quantum Mechanics is QM-as-QM and everything else is everything else!

"The one thing to say about art is that it is *one thing.* Art is *art-as-art* and everything else is everything else." (Ad Reinhardt)

A Vision of Quantum Mechanics?



Twelve Rules for a
New Academy

No texture
No brushwork
No drawing
No forms
No design
No color
No light
No space
No time
No size or scale
No movement
No object

Ad Reinhardt, '53

Ad Reinhardt (American, 1913–1967)
Medium: Oil on canvas

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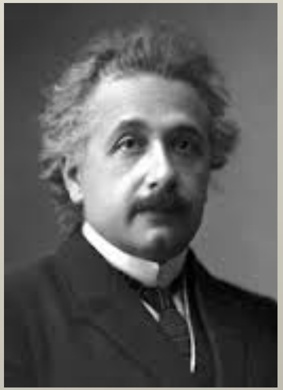
"Alle Naturwissenschaft ist auf die Voraussetzung der vollständigen kausalen Verknüpfung jeglichen Geschehens begründet."

(Albert Einstein, 1910)

Well – is it?



Attempting to answer this question is the general theme of this lecture. We will analyze the seemingly puzzling features of quantum (probability) theory.

Let's get started!

1. Is Quantum Probability Th. ≡ Class. Probability Theory?

And – if not – how does it differ?

<u>Class. (topol.) dynamical syst.</u>:

M: (cp. topol.) state space;
σ-alg., Σ, of Borel sets.

$\mathcal{A} := C(M)$

$\{\tau_{t,s}\}_{t,s \in \mathbb{R}} \subset Aut(\mathcal{A})$: time evol.
$\overset{1-1}{\longleftrightarrow}$ homeos $\{\phi_{t,s}\}_{t,s\in\mathbb{R}}$ of M

$\omega, \rho, \cdots$: States = prob. meas. on (M,Σ).

π: meas. class; $\mathcal{A}^{\pi} := L^{\infty}(M,\pi)$

$\Pi^{(i)} := \chi_{\Omega_i}(\cdot)$, $\Omega_i \in \Sigma$ ($\neq$ null set)

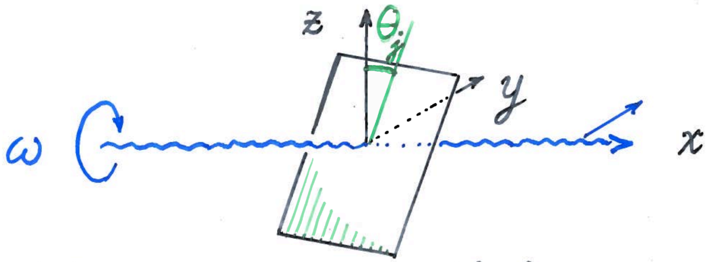
$\omega \in \pi$: state

$$Prob_{\omega}\{\Pi^{(0)}_{t_0}, \cdots, \Pi^{(n)}_{t_n}\} := \int_M d\omega(\xi) \prod_{i=0}^{n} \chi_{\Omega_i}(\phi_{t_i,t_*}(\xi)) \qquad (1)$$

ω pure $\Leftrightarrow$ $\omega = \delta_{\xi_*}$, $\xi_* \in M$
$\rightarrow$ 0–1 laws
etc.

Example of quantum syst.

Beam of (monochromatic) light
≃ beam of photons
w. $n+1$ polarization filters



After passing j^{th} filter, photon pol. in dir. $\theta_j := \frac{j\pi}{2n}$

Initially, beam circ. pol.

$\Pi_+^{(j)}$: photon passes through j^{th} filter;

$\Pi_-^{(j)}$: photon absorbed in j^{th} filter.

$$\mathrm{Prob}_\omega\{\Pi_+^{(\theta)} \mid \Pi_+^{(\varphi)}\} = \cos^2(\theta-\varphi),$$

$$\mathrm{Prob}_\omega\{\Pi_-^{(\theta)} \mid \Pi_+^{(\varphi)}\} = \sin^2(\theta-\varphi),$$

$$\mathrm{Prob}_\omega\{\Pi_\pm^{(0)}\} = \frac{1}{2}.$$

Thus,

$$\mathrm{Prob}_\omega\{\Pi_+^{(0)}, \dots, \Pi_+^{(n)}\} = \frac{1}{2}\left(\cos\left(\frac{\pi}{2n}\right)\right)^{2n} \xrightarrow[n\to\infty]{} \frac{1}{2}$$

$$\mathrm{Prob}_\omega\{\Pi_\pm^{(0)}, \Pi_+^{(n)}\} = 0$$

If syst. were class. dyn. syst.

$$\left(\tfrac{1}{2}=\right)\mathrm{Prob}_\omega\{\Pi_+^{(0)}, \dots, \Pi_+^{(n)}\} \leq \mathrm{Prob}_\omega\{\Pi_+^{(0)}, \Pi_+^{(n)}\} \ (=0)$$

because $\Pi_+^{(j)} + \Pi_-^{(j)} = 1$, $\Pi_\pm^{(j)} \geq 0$.

Interference!

$\Rightarrow \mathrm{Prob}_\omega$ not given by (1)!

More sophisticated arguments

Kochen–Specker, Bell's <

K–S: Measure spin $s=1$.

$$S_x^2+S_y^2+S_z^2=s(s+1)=2,$$

$$(S_i^2)^2=S_i^2,\ [S_i^2,S_j^2]=0,\ \forall(\vec{e}_x,\vec{e}_y,\vec{e}_z)$$

If $\{S_i^2\}$ were class. rv's then 2 out of $\{S_x^2,S_y^2,S_z^2\}$ are $=1$, remaining one $=0$, $\forall(\vec{e}_x,\vec{e}_y,\vec{e}_z)$

$\longrightarrow$ impossible!

Bell: 2 indep. spins, $s=\frac{1}{2}$.

Class. rv:

$$|\langle\sigma_{\underline{1}}\cdot\sigma_{\underline{2}}\rangle+\langle\sigma_{\underline{1}}\cdot\sigma_{\underline{3}}\rangle+\langle\sigma_{\underline{4}}\cdot\sigma_{\underline{2}}\rangle-\langle\sigma_{\underline{4}}\cdot\sigma_{\underline{3}}\rangle|\leqslant 2$$

$\forall\ \underline{1},\underline{2},\underline{3},\underline{4}$.

QM:

$$\max|\langle\sigma_{\underline{1}}\cdot\sigma_{\underline{2}}\rangle+\langle\sigma_{\underline{1}}\cdot\sigma_{\underline{3}}\rangle+\langle\sigma_{\underline{4}}\cdot\sigma_{\underline{2}}\rangle-\langle\sigma_{\underline{4}}\cdot\sigma_{\underline{3}}\rangle|=2\sqrt{2}$$

2. What is a Quantum Dynamical System?

$\mathcal{A}=C(M)\longmapsto\mathcal{A}$: NC C^*-alg.

w. $\{\tau_{t,s}\}_{t,s\in\mathbb{R}}\subset{}^*\mathrm{Aut}(\mathcal{A})$,

$\tau_{t,s}\circ\tau_{s,u}=\tau_{t,u}$: time evolution

"autonomous": $\tau_{t,s}=\tau_{t-s}$.

$\omega,\rho,\cdots$: states on $\mathcal{A}$ prescribed at time t_*.

Time evol. in Heisenberg picture: $a(t):=\tau_{t,t_*}(a)$, (2) $a\in\mathcal{A}$.

$\mathcal{P}$ subset of $\mathcal{A}$ closed under *

$\langle \mathcal{P} \rangle := C^*$-algebra in $\mathcal{A}$ generated by $\mathcal{P}$.

$(\mathcal{A}, \omega) \to$ rep. π_ω of $\mathcal{A}$ on Hilbert space $\mathcal{H}_\omega$, $\Omega \in \mathcal{H}_\omega$: cyclic for $\pi_\omega(\mathcal{A})$ s.t.

(3) $\omega(a) = \langle \Omega, \pi_\omega(a)\Omega \rangle$, $\forall a \in \mathcal{A}$.

(GNS construction)

π : Rep. of $\mathcal{A}$ on some $\mathcal{H}$

$\mathcal{A}^\pi$: weak* closure of $\mathcal{A}$ in $B(\mathcal{H})$

a von Neumann alg.

$\mathcal{S}^\pi$: Normal states on $\mathcal{A}^\pi$

$(\mathcal{A}, \pi, \mathcal{A}^\pi) \leftrightarrow$ measure class

$\mathcal{S}^\pi \leftrightarrow$ {probability measures}

Let $a = a^* \in \mathcal{A}$ have finite spec:

$$\pi(a) = \sum_{i=1}^{k} \underbrace{\alpha_i}_{\text{e.v.'s}} \underbrace{\Pi_i^{(a)}}_{\text{spec. proj.} \in \mathcal{A}^\pi}, \qquad (4)$$

$$\Pi_i^{(a)} \Pi_j^{(a)} = \delta_{ij} \Pi_i^{(a)}, \sum_{i=1}^{k} \Pi_i^{(a)} = \mathbb{1}. \qquad (5)$$

For $\mathcal{B}$ *subalg. of $\mathcal{A}$, define

$$\mathcal{B}' \cap \mathcal{A} := \{ a \in \mathcal{A} \mid [a, b] = 0, \forall b \in \mathcal{B} \}$$

Back to Physics

Duality between "observables" and "indeterminates":

"Observables" or "potential props."

$\leftrightarrow$ ops. in

$$\mathrm{O} := \{ a_i = a_i^* \in \mathcal{A} \mid i \in I \} \qquad (6)$$

with: $a \in \mathcal{O}$, f cont. $\mathbb{R}$-valued function on $\mathbb{R} \Rightarrow f(a) \in \mathcal{O}$.

Choose a rep. π of $\mathcal{A}$; let $(\mathcal{A}^\pi, \mathcal{S}^\pi)$ be as above.

Definition.

- $\mathcal{E}_{\geq t} := \langle \{ \prod_{i=1}^{n} a_i(t_i) \,|\, a_i \in \mathcal{O}, t_i \geq t \} \rangle^\pi$

  v.N. alg. of "possible events" at times $\geq t$.

- $\mathcal{E} := \left(\bigvee_{t \in \mathbb{R}} \mathcal{E}_{\geq t} \right)^{-\pi}$

  "Indeterminates":

$$\mathcal{J} := \mathcal{E}' \cap \mathcal{A}^\pi \qquad (7)$$

Fundamental Duality Principle ($\to$ entanglement $\mathcal{E}$):

(8) $a(t) = \tau_{t,t_*}(a)$, $a \in \mathcal{O}$, $t \in \mathbb{R}$, may corresp. to meas./event at time t only if $\mathcal{J}$ non-trivial, containing (sub)alg. $\simeq \mathcal{E}$.

$$\Downarrow$$

$$\mathcal{A}^\pi \underset{(8)}{\supsetneqq} \mathcal{E} \supseteq \mathcal{E}_{\geq t} \underset{\uparrow}{\supsetneqq} \mathcal{E}_{\geq t'} \supseteq \{\mathbb{C}\mathbb{1}\} \qquad (9)$$

"Loss of information" ($\mathcal{L}o\mathcal{J}$)

As an "initial condition" choose a state $\omega \in \mathcal{S}^\pi$.

"Everybody" agrees on following "Postulate": If $a \in \mathcal{O}$, as in (4), (5), is observed/

measured at time t then state right after meas. of a given by

$$\simeq \sum_{i=1}^{k} p_i \omega_i \qquad (10)$$

where $p_i = \omega\left(\Pi_i^{(a(t))}\right) \qquad (11)$

(Born)

and meas. of a at time $t+0$ in state ω_i yields value α_i with certainty.

This "Copenhagen postulate" calls for an explanation!

$\mathfrak{M}$: von Neumann algebra;

ω : normal state on $\mathfrak{M}$.

Definition. $\{a,\omega\}_{\mathfrak{M}}$, $a \in \mathfrak{M}$, is bd. linear functional on $\mathfrak{M}$:

$$(12) \quad \{a,\omega\}_{\mathfrak{M}}(b) := \omega([a,b]), \; b \in \mathfrak{M}.$$

Lemma 1. Let $a = a^* \in \mathfrak{M}$ be as in (4), (5). Then

$$\|\{a,\omega\}_{\mathfrak{M}}\| < \varepsilon \Leftrightarrow \omega(b) = \sum_{i=1}^{k} p_i \omega_i(b) + O(\varepsilon\|b\|)$$

where

$$p_i = \omega(\Pi_i^{(a)}) \text{ and, for } p_i \neq 0,$$

$$\omega_i(b) = p_i^{-1} \omega(\Pi_i^{(a)} b \Pi_i^{(a)})$$

Definition.

$$\mathcal{E}_{\mathfrak{M}}^{\omega} := \{a \in \mathfrak{M} \mid \{a,\omega\}_{\mathfrak{M}} = 0\}$$

"stabilizer" of ω

$\mathcal{Z}^{\omega}_{\mathfrak{M}}$: center of $\mathcal{C}^{\omega}_{\mathfrak{M}}$; (ab.).

Lemma 2.

(i) ω is a normalized trace on $\mathcal{C}^{\omega}_{\mathfrak{M}}$

(ii) $\mathcal{C}^{\omega}_{\mathfrak{M}} = \int^{\oplus}_{\Lambda_\omega} M_{n_\lambda}(\mathbb{C}) \oplus$ "type II_1's"

$1 \leq n_\lambda < \infty$, $\forall \lambda \in \Lambda_\omega$.

($\nearrow$ classification of $\mathfrak{M}$'s)

(iii) (If, e.g., ω separating on $\mathfrak{M}$)

$\exists$ "conditional expectation"

$$\varepsilon : \mathfrak{M} \to (\mathcal{C}^{\omega}_{\mathfrak{M}} \to) \mathcal{Z}^{\omega}_{\mathfrak{M}}, \qquad (13)$$

satisfying

$$\varepsilon(x^* x) \geq \varepsilon(x)^* \varepsilon(x), \ \forall x \in \mathfrak{M},$$

$$\varepsilon(a x b) = a\,\varepsilon(x)\,b, \ \forall x \in \mathfrak{M},$$

$$\forall a, b \in \mathcal{Z}^{\omega}_{\mathfrak{M}} / \mathcal{C}^{\omega}_{\mathfrak{M}}.$$

Application.

$$\mathcal{C}^{\omega} := \mathcal{C}^{\omega}_{\mathcal{E}}, \ \mathcal{C}^{\omega}_{\geq t} := \mathcal{C}^{\omega}_{\mathcal{E}_{\geq t}}, \ \mathcal{Z}^{\omega}_{\geq t} := \mathcal{Z}^{\omega}_{\mathcal{E}_{\geq t}},$$

$$\varepsilon_{\geq t} : \mathcal{E}_{\geq t} \to \mathcal{Z}^{\omega}_{\geq t}.$$

Definition. (Variance of a)

For $a \in \mathcal{E}_{\geq t}$, define

$$\Delta^{\omega}_{t} a := \sqrt{\omega([a - \varepsilon_{\geq t}(a)]^2)}$$

Definition. ("Empirical props.")

Let $a \in \mathcal{O}$ satisfy (4), (5). We say that a is an "empirical property at time $\approx t$", given a resolution $\delta \geq 0$, iff

$$\Delta^{\omega}_{t} a(t) \leq \delta \qquad (14)$$

- If $\Delta^{\omega}_{t}\, a(t) \leqslant \delta \ll 1$ then $a(t)$ is "close" to an op. in $\mathcal{Z}^{\omega}_{\geqslant t}$, and

$$\omega(b) = \sum_{i=1}^{k} \omega\left(\Pi_i^{(a(t))}\, b\, \Pi_i^{(a(t))}\right) + O(\delta \|b\|), \qquad (15)$$

$\forall b \in \mathcal{E}_{\geqslant t}$.

- $\omega|_{\mathcal{E}_{\geqslant t}} =: \rho_t$ (traj. of states)

If $\mathcal{E}_{\geqslant t} \simeq B(\mathcal{H}^{\pi})$, $\forall t \in \mathbb{R}$, then ρ_t's are density matrices; t-dep. approx. by Lindblad evolution (↗ DeRoeck & J.F., ...).

(15) $\Rightarrow S(\rho_t) > 0$ even if $S(\omega|_{\mathcal{E}}) = 0$; i.g. ($\mathcal{L}o\mathcal{T}$ & $\mathcal{E}$)

- $\Delta^{\omega}_{t}\, a(t) \leqslant \delta \ll 1 \Rightarrow$

$$a(t)\big|_{\mathrm{Ran}\,\rho_t} \approx F(\rho_t) \qquad (16)$$

Formula (~L-S-Wigner, ↗(1)) for Prob of "consistent histories" assoc. w. family of "empirical properties".

- $\mathcal{Z}^{\omega}$ (center of $\mathcal{C}^{\omega}_{\mathcal{E}}$): abelian alg. of "class. events", given state ω.

$\mathcal{Z}^{\omega}_{\geqslant t}$ ab. alg. of "empirical props." at times $\geqslant t$.

→ Theory of "von Neumann measurements".

3. Non-Demolition Measurements & the Emergence of Facts

S : system (cavity w. e.m. field)

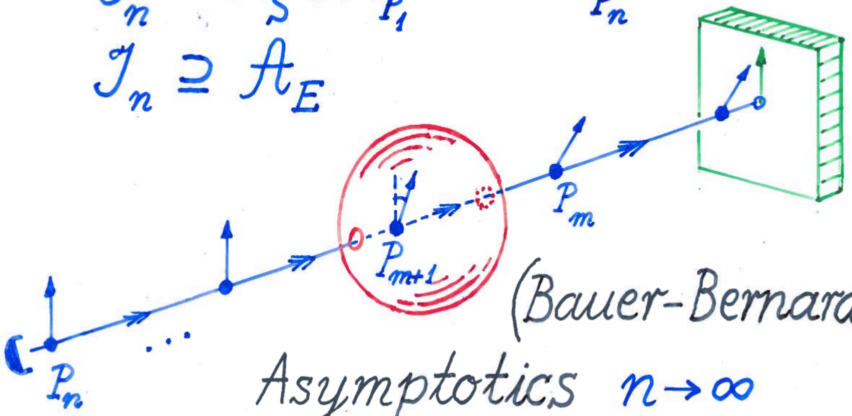
P : probe (exc. atom prep. in superpos. of 2 exc. states)

E : filter for probe.

$$\mathcal{A}_n := \mathcal{A}_S \otimes \mathcal{A}_{P_1} \otimes \cdots \otimes \mathcal{A}_{P_n} \otimes \mathcal{A}_E$$

$$\mathcal{O}_n \subset \mathcal{A}_S \otimes \mathcal{A}_{P_1} \otimes \cdots \otimes \mathcal{A}_{P_n}$$

$$\mathcal{I}_n \supseteq \mathcal{A}_E$$



(Bauer-Bernard)

Asymptotics $n \to \infty$

Want to measure pot. property (e.g., photon #) of S corresp. to $a \otimes 1_1 \otimes \cdots \otimes 1_n \otimes 1_E \in \mathcal{O}_n$, with

$$a = a^* = \sum_{i=1}^{k} \alpha_i \, \Pi_i^{(a)},$$

using "non-demolition exp.", (as $n \to \infty$).

$$\mathcal{A}_S \ni e_{ij} : \operatorname{Ran} \Pi_j^{(a)} \to \operatorname{Ran} \Pi_i^{(a)},$$

$$\Pi_{i'}^{(a)} e_{ij} \Pi_{j'}^{(a)} = \delta_{i'i} \delta_{jj'} e_{ij}.$$

Time evolution: During time int. $[mT, (m+1)T)$, only P_{m+1} interacts with S; after time $(m+1)T$, E does v. N. measurement of

$$b^{(m+1)} := \mathbb{1}_1 \otimes \cdots \otimes \underset{\substack{\uparrow \\ m+1}}{b} \otimes \cdots \otimes \mathbb{1}_n \otimes \mathbb{1}_E \in \mathcal{O}_n$$

where $b = b^* \in \mathcal{A}_{P_{m+1}} \simeq M_N(\mathbb{C})$, $\forall m$

$$\Rightarrow \; b = \sum_{\xi=1}^{N} \beta_\xi \, \pi_\xi \;, \left(\pi_\xi = |\xi\rangle\langle\xi|\right).$$

$$\tau_{(m+1)T, mT}\left(e_{ij} \otimes b^{(m+1)}\right)$$

$$= e_{ij} \otimes \mathbb{1}_1 \otimes \cdots \otimes U^*_{\alpha_i} \, b \, U_{\alpha_j} \otimes \cdots \otimes \mathbb{1}_n \otimes \mathbb{1}_E,$$

$\{U_{\alpha_i}\}_{i=1}^{k}$ are $N \times N$ unitary mat.

Time evol. trivial for $P_{m+2}, \dots, P_n$; interaction betw. P_j & E, for $j \leq m$ ($\rightarrow$ v. N. meas. of $b^{(1)}, \dots, b^{(m)}$) on $[mT, (m+1)T)$.

Initial state:

$$\Psi^{(n)} := \omega_S \otimes \underset{1}{\psi} \otimes \cdots \otimes \underset{n}{\psi} \otimes \varphi_E$$

After passage of $\ell \gg 1$ probes

$$(17) \quad \Psi^{(n)}\Big|_{\mathcal{E}_{\geq \ell T}} \simeq \sum_{i=1}^{k} p_i \, \omega_i \otimes \underset{\ell+1}{\psi} \otimes \cdots \otimes \underset{n}{\psi} \otimes \varphi_E^{(i)}$$

(first ℓ probes "lost"), where

$$\omega_i = p_i^{-1} \omega_S\left(\Pi_i^{(a)} (\cdot) \Pi_i^{(a)}\right) \quad (p_i \neq 0)$$

$$p_i = \omega_S\left(\Pi_i^{(a)}\right)$$

(exp.) "decoherence"

Thus, may assume that

$$\Psi^{(n)} = \sum_{i=1}^{k} p_i \, \Psi_i^{(n)},$$

$$\Psi_i^{(n)} = \omega_i \otimes \psi \otimes \cdots \otimes \psi \otimes \varphi_E^{(i)}$$

ω_i : eigenstate of a w.e.v. α_i.

$$p(\xi|i) := \psi(U^*_{\alpha_i} \pi_\xi U_{\alpha_i})$$

$$\underline{\xi}_r := \{\pi_{\xi_1}, \cdots, \pi_{\xi_r}\} \equiv \{\underline{\xi}_{r-1}, \xi_r\}$$

"probe histories"

$$\mu(\underline{\xi}_r|i) := Prob_{\Psi_i^{(n)}}\{\pi_{\xi_1}, \cdots, \pi_{\xi_r}\}$$

$$= \prod_{s=1}^{r} p(\xi_s|i)$$

$$\mu(\underline{\xi}_r) := \sum_{i=1}^{k} p_i \mu(\underline{\xi}_r|i)$$

$p^{(r)}(i|\underline{\xi}_r)$ = cond. prob. of $\Pi_i^{(a)}$, given $\underline{\xi}_r$.

$$= p_i \frac{\mu(\underline{\xi}_r|i)}{\mu(\underline{\xi}_r)} \qquad (18)$$

(i) $0 \leq p^{(r)}(i|\underline{\xi}_r) \leq 1$, &

$$\sum_{i=1}^{k} p^{(r)}(i|\underline{\xi}_r) = 1.$$

(ii)

$$p^{(r)}(i|\underline{\xi}_r) = p^{(r-1)}(i|\underline{\xi}_{r-1}) \frac{p(\xi_r|i)}{\sum_{j=1}^{k} p^{(r-1)}(j|\underline{\xi}_{r-1}) p(\xi_r|j)}$$

$$\Downarrow$$

(iii) $\mathbb{E}_{\xi_r} p^{(r)}(i|\underline{\xi}_r) = p^{(r-1)}(i|\underline{\xi}_{r-1})$

(i), (iii) $\Rightarrow \{p^{(r)}(i|\underline{\xi}_r) \,|\, \alpha_i$ e.v. of $a\}$ are bounded martingales.

MCThm.: $p^{(r)}(i|\underline{\xi}) \underset{r\to\infty}{\longrightarrow} p^{(\infty)}(i|\underline{\xi})$

(ii) $\Rightarrow$

$$p^{(\infty)}(i|\underline{\xi}) = p^{(\infty)}(i|\underline{\xi}) \frac{p(\xi_\infty|i)}{\sum_{j=1}^{k} p^{(\infty)}(j|\underline{\xi}) p(\xi_\infty|j)}$$

non-deg. cond. $\Rightarrow$ $p^{(\infty)}(i|\underline{\xi}) = \delta_{i, i_*(\underline{\xi})}$, a.e. $\underline{\xi}$,

! with $E_{\underline{\xi}}\, p^{(\infty)}(i|\cdot) = p_i$.

4. Applications

(1) Foundations of “quantum theory of experiments”
(2) Experiments by S. Haroche et al. (cavity QED).
(3) Emergence of a particle track when particle is repeatedly illuminated by (independent) light pulses and scattered light is measured.

Etc.

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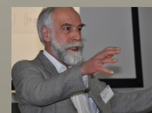
Effective quantum dynamics in all such problems can be described in terms of a (small perturbation around a) *quantum Markov process*, i.e., *Lindblad dynamics.* This theme is developed in joint work with *De Roeck*; (on QBM).

"In all my films, I have been faithful to these suspension points in the conclusions. Besides, I have never written the word 'END' on the screen."
(Federico Fellini)

"Everyone wants to understand art (physics). Why don't we try to understand the song of a bird? Why do we love the night, the flowers, everything around us, without trying to understand them? But in the case of a painting (result in physics), people think they have to understand." (Pablo Picasso)

Thank you for listening!



“What is better than - when the work is done, with a mind free of all worries and tired from efforts to the benefit of others – to return to our home and lie down to rest on the bed we so desired?”

(Catullus, as quoted in the Villa Cimbrone)